

Localization in periodically modulated speckle potentials

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Disorder induces Anderson localization of lattice eigenstates and drastically alters its transport properties. In the original 1D Anderson set-up, the addition of a periodic driving increases, in a certain range of the driving's frequency and amplitude, localization length of the Floquet eigenstates. We go beyond the uncorrelated disorder model to address the experimentally relevant spatial correlations which bring an effective mobility edge in the lattice energy spectrum. We find that in the regime of weak driving the resulting Floquet states undergo resonant hybridization but remain localized in the basis of stationary Hamiltonian. As the amplitude of the driving is increased further, the Floquet states start to delocalize. In the original basis, this corresponds to two opposite trends for the strongly localized and quasi-extended bands: the former become less compact, while the latter more. Ultimately, a strong driving tends to equate their localization properties of the eigenstates belonging to the different bands.

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Anderson localization in disordered systems is a fundamental phenomenon that is still posing new puzzles and bringing new surprises [1–3]. The original problem of non-interacting quantum particles [4] was studied thoroughly and has been placed in a much broader context, coming to experimental observations with matter [5–8], electromagnetic [9], and acoustic waves [10].

The effect of periodic modulations on the localization also received considerable attention. It was found that the localization length increases under the low frequency driving (though non-monotonously with the driving amplitude) and decreases in the opposite limit [11]. The increase of the localization length was attributed to the induced interaction between the particle path channels, with those characterized by weakest localization making a dominant contribution. In contrast, the high-frequency driving diminishes time-averaged hopping amplitudes [11–13] and enhance the localization, an effect reminiscent of the dynamic localization [14, 15]. Recently, it has been shown that the multi-frequency driving can substantially increase the localization length in the low frequency region, with two different scaling regimes for weak and strong modulations, although the localization length remains finite for a finite number of frequency components [16]. Noteworthy, a complete delocalization can be achieved with driven quasi-periodic potentials [17].

The existing results, however, address the original Anderson set-up, with on-site lattice energies being random and *uncorrelated* variables. At the same time, the presence of *correlations* is inherent to the optical speckle potentials, used in the experiments with atomic Bose-Einstein condensates [18]. Importantly, a finite correlation length leads to emergence of an effective mobility edge even in 1D disordered lattices, separating the bands

with localization lengths which differ by orders of magnitude [19–21].

A perspective of introducing periodic driving into systems with correlated disorder immediately evokes a set of intriguing questions. Does it affect the emerged quasi-extended states differently than in the case of uncorrelated disorder? Can it induce a coupling across the mobility edge? Can the co-existence of the strongly localized and quasi-extended states change the modulated localization picture?

In this paper we study the fate of localization in a periodically and slowly driven 1D speckle potential. The results of a perturbative analysis and computational studies indicate that the Floquet states remain localized in the stationary Hamiltonian basis under weak driving, thus inheriting the traits of stationary eigenstates. A stronger driving leads to a resonant hybridization of Floquet states, and the ultimate ‘smearing’ of the Floquet states in the eigenstate basis. In the direct space it corresponds to the opposite changes for the two kinds of states. Namely, strongly localized states weaken their localization, while quasi-extended states get more localized. Both, however, loose compactness and become sparser. Finally, the majority of Floquet states homogenizes.

We consider the dynamics of a quantum particle in a speckle potential described by a tight-binding Hamiltonian under time-periodic modulations,

$$H(t) = \sum_j \epsilon_j (1 + A \cos \omega t) b_j^\dagger b_j + \frac{1}{2} (b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j), \quad (1)$$

where b_j and b_j^\dagger are the annihilation and creation operators of a particle at site j , A and ω are the driving amplitude and frequency, and $\epsilon_j > 0$ are correlated random on-site energies representing the so-called “blue-detuned”

potential [18]. In the particle number basis, $|j\rangle \equiv b_j^\dagger|0\rangle$, and with $m = \hbar = 1$, the Schrödinger equation assumes the form

$$i\dot{\psi}_j = \epsilon_j (1 + A \cos \omega t) \psi_j + \frac{1}{2} (\psi_{j+1} + \psi_{j-1}). \quad (2)$$

For a finite lattice of the length N , the set of on-site energies can be obtained by a mathematical procedure that serves a discrete analog of an optical speckle [18]. Namely, one keeps D (“aperture” parameter) out of N Fourier components for a realization of a complex Gaussian variable, and assigns the squared absolute values to ϵ_j . The resulting random potential follows Rayleigh distribution with mean $\langle \epsilon \rangle = I_0$ and correlation length $\xi \approx 0.88N/D$.

The key property of the stationary case, $A = 0$, is the presence of an effective mobility edge in the spectrum, estimated as [19, 20] $E_M = I_0 + (0.88\pi/\xi)^2/2$. It separates the strongly and quasi-extended states, Fig. 1(a). The localization length of the latter is one or two orders of magnitude larger than that of the former, so that and in finite systems the quasi-extended states can be treated as de-localized [21]. At higher energies the strong localization is restored as “semi-classical” localization.

Dynamics of the driven system can be fully evaluated in terms of the so-called “Floquet eigenstates” [22–24]. They are the eigenstates of the unitary Floquet propagator $U_T = \mathcal{T} \exp \left[-\frac{i}{\hbar} \int_0^T H(\tau) d\tau \right]$, where \mathcal{T} is the time-ordering operator. The particular structure of the Floquet propagator, and thus the localization properties of Floquet states, depend on the parameters of the driving. This dependence is the main objective of our study.

Considerable insight can be obtained by expressing the Floquet states in the basis of the stationary Hamiltonian, $\{\mathbf{z}^{(l)}\}$, such that $H\mathbf{z}^{(l)} = \lambda_l \mathbf{z}^{(l)}$, $A = 0$. By using expansion $\psi = \sum_l \varphi_l \mathbf{z}^{(l)}$, Eq.(2) can be rewritten as

$$i\dot{\varphi}_k = \lambda_k \varphi_k + A \cos \omega t \sum_{k'} J_{k,k'} \varphi_{k'}, \quad (3)$$

where $J_{k,k'} = \sum_j \epsilon_j z_j^{(k)} z_j^{(k')}$ are the driving-induced overlap coefficients.

The Floquet states $F_k^{(l)}(t)$ have the form

$$F_k^{(l)}(t) = \Phi_k^{(l)}(t) \exp(-is_l t), \quad (4)$$

where $\Phi_k^{(l)}(t)$ are $T = 2\pi/\omega$ -periodic functions, eigenstates of the Floquet propagator, and s_l are quasi-energies. Evidently, in the absence of driving, $A = 0$, the Floquet states correspond to the stationary states, $\Phi_k^{(l)} = \delta_{k,l}$, and the quasi-energies are given by the respective eigenvalues (energies), $s_l = \lambda_l$.

In this picture, the Floquet states are results of driving-induced cross-breeding between the stationary counterparts. Using the standard perturbation theory, we expand $\Phi_k(t)$ into Fourier series and express Floquet states

as

$$F_k(t) = \sum_n \phi_k^{(n)} \exp(-in\omega t) \exp(-ist). \quad (5)$$

Substituting (5) into (3), we get

$$s\phi_k^{(n)} = (\lambda_k - n\omega)\phi_k^{(n)} + \frac{1}{2}A \sum_{k'} J_{k,k'} \left(\phi_{k'}^{(n-1)} + \phi_{k'}^{(n+1)} \right). \quad (6)$$

Taking the l -th Floquet state for $A = 0$ as a zero-order approximation, $\phi_k^{(0)} = \delta_{k,l}$ and $s = \lambda_l$, for the first order we obtain non-zero amplitudes

$$\phi_k^{(\pm 1)} = \frac{AJ_{k,l}}{2(\lambda_l - \lambda_k \pm \omega)}. \quad (7)$$

The net norm of the first order corrections is, therefore,

$$\|\Delta\phi^{(l)}\| = \frac{1}{4}A^2 \sum_k \frac{J_{k,l}^2}{(\lambda_l - \lambda_k \pm \omega)^2}, \quad (8)$$

and provides with the necessary validity condition $\|\Delta\phi^{(l)}\| \ll 1$.

Note that the higher order resonances will also be induced in chain, until the multiple of the driving frequency stays within the spectrum bound.

Clearly, the degree of hybridization between two Floquet states depends on both the overlap coefficient between the states, $J_{k,l}$, and the distance to the resonance, $|\lambda_k - \lambda_l| \approx n\omega$, $n \in \mathbb{N}$. To secure the latter possibility, the driving has to be *slow* so that its frequency does not exceed the characteristic spectrum width. Furthermore, it can be expected that most of the strongly localized states would not interact significantly, as both the near-resonance and overlap conditions would rarely be satisfied at the same time.

A localized state occupies a certain localization volume $V_L \ll N$, and normalization requires that the non-zero elements scale as $|z_l| \sim V_L^{-1/2}$. We assume that the signatures of the eigenstates elements $z_j^{(k)}$ and $z_j^{(l)}$ are not correlated, and obtain $|J_{k,l}| \sim V_L^{-1/2} \Theta(k,l)$, where $\Theta(k,l) = 1$ if the eigenstates overlap, and 0 otherwise. As the probability of the latter scales as $\sim V_L/N$, we finally obtain $|J_{k,l}| \sim V_L^{1/2}/N$. In turn, the contribution of the frequency denominator can be estimated as follows. Since the density of states scales linearly with N , the eigenstate spacing behaves as $\sim 1/N$, and the squared resonance mismatch in the denominator yields $\sim 1/N^2$. Thus we arrive at $\|\Delta\phi^{(l)}\| \sim A^2 V_L$. Thus, the first corrections due to driving-induced interactions between strongly localized states keep finite, which is consistent with the moderate influence of driving, found in the Anderson (uncorrelated) disorder case [11–13].

Appearance of the effective mobility edge and quasi-extended band due to correlations in disorder, enriches the picture. Indeed, many quasi-extended modes would

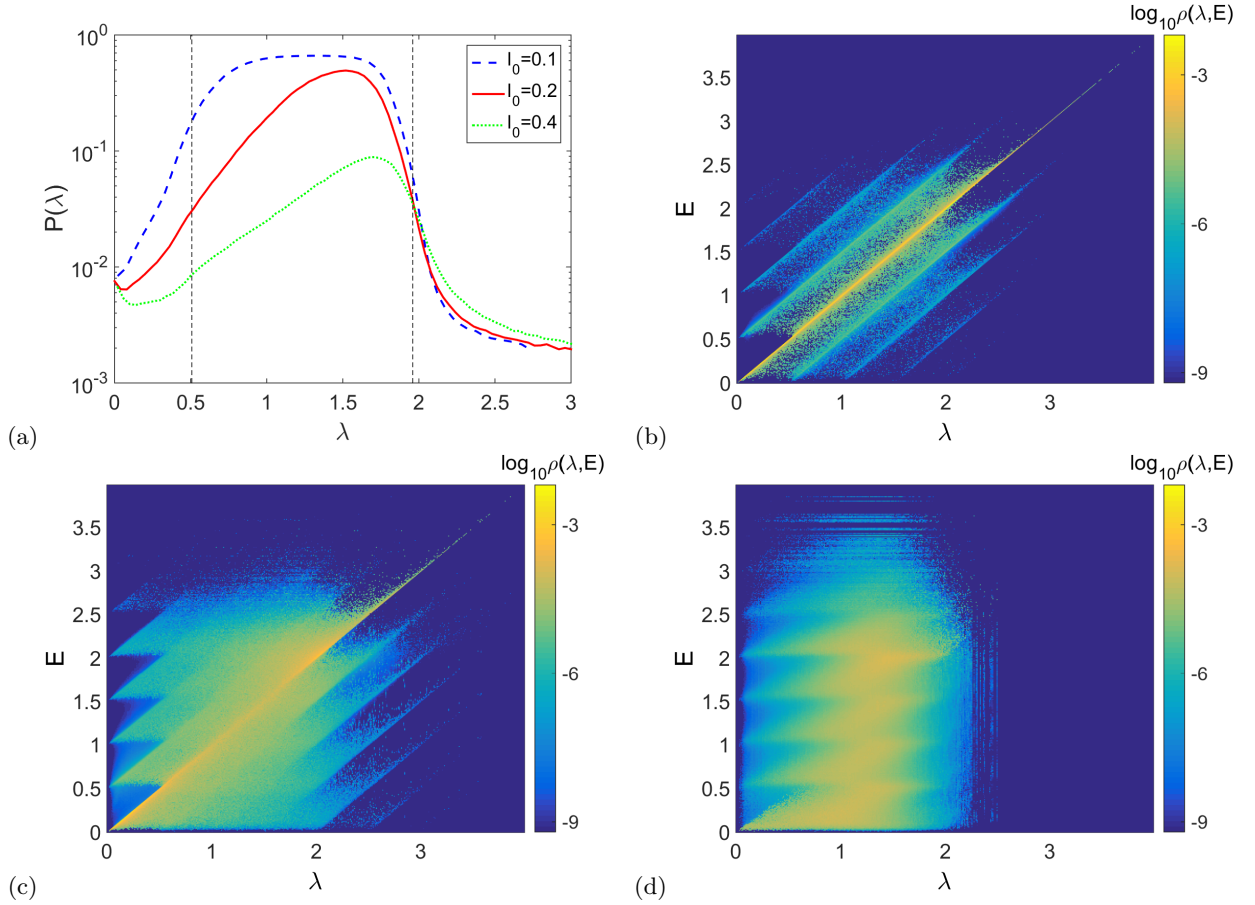


FIG. 1. (a) Localization in the stationary lattice with correlated disorder: average participation number of eigenstates normalized by the system size, $P = 1/(N \sum |\psi_j|^4)$, as functions of energy, λ , for different disorder strength I_0 . Vertical dashed lines indicate the band of quasi-extended states: effective mobility edge (left) and the border with “semi-classical” strong localization (right); (b)-(d) Floquet states in the stationary basis, $\rho(\lambda, E)$, for $I_0 = 0.2, \omega = 0.5$ and different amplitude of driving, $A = 0.01, 0.1$ and 1.0 , respectively. Here $N = 1600$ and $D = 400$. The results correspond to the averaging over $N_r = 100$ disorder realizations..

fall within the same localization volume $V_E \gg V_L$ and interact efficiently. Moreover, they may couple across the mobility edge to the strongly localized ones, and affect the latter. Repeating the above arguments, for the overlap integrals between quasi-extended states or between such a state and a strongly localized one, we obtain $\|\Delta\phi^{(l)}\| \sim A^2 V_E$. Given $V_E/V_L \sim 10^1 \dots 10^2$, there follows a dramatically stronger effect of driving on localization in speckle potentials, as compared to uncorrelated disorder.

We now turn to the results of computational studies. The Floquet states are found by numerical integration of Eq. (2) by the high-precision Magnus-Chebyshev scheme over a period of driving [25], with 200 integration steps per period. We choose $N/D = 4$ so that the correlation length $\xi \approx 3.5$ and make the core simulations for $N = 1600 \approx 450\xi$. The disorder strength is set to $I_0 = 0.2$ so that one obtains $E_M \approx 0.5$, ensuring a substantial part of the states of both kinds; see Fig.1(a). Finally, zero

boundary conditions are assumed.

First, we study the structure of Floquet states in the eigenbasis of the stationary system. We calculate the period-averaged norm [29] of each element of the Floquet state, $\|F_k^{(l)}\| = \frac{1}{T} \int_0^T |F_k^{(l)}(t)|^2 dt$, and its average energy, $E_l = \sum_k \lambda_k \|F_k^{(l)}\|$. We define the probability density function

$$\rho(\lambda, E) = \lim_{\Delta\lambda, \Delta E \rightarrow 0} \frac{\langle \|F_k^{(l)}\| \rangle_{\lambda_k \in [\lambda, \lambda + \Delta\lambda], E_l \in [E, E + \Delta E]}}{\Delta\lambda \Delta E}, \quad (9)$$

which, additionally averaged over disorder realizations, essentially gives the weight of stationary eigenstates with the energy λ in the set of Floquet states with the average energy E .

Figure s1(b)-(d) present the modification in the Floquet states under slow driving of the frequency $\omega = 0.5$ (less than characteristic spectrum width). For weak driving, $A = 0.01$, the Floquet states are only slightly

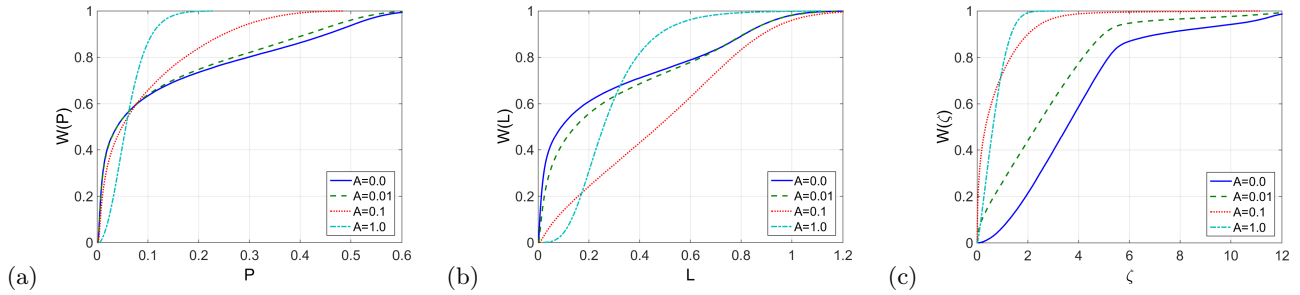


FIG. 2. Localization properties of Floquet states as functions of driving amplitude A : integral distributions of (a) normalized participation number P ; (b) normalized effective width L ; and (c) compactness index, ζ . Here $\omega = 0.5$, $I_0 = 0.2$, $N = 1600$, $D = 400$. The results correspond to the averaging over $N_r = 100$ disorder realizations.

perturbed zero-driving solutions, with prominent distortions, however, when near-resonance condition holds, see Fig. 1(b). The main effect is observed for the quasi-extended states. A stronger driving, $A = 0.1$, induces a progressive resonant hybridization of states, including the strongly localized ones, see Fig. 1(c). As the driving amplitude grows further to $A = 1.0$, the Floquet states become broadly smeared over the stationary basis and completely loose the initial form, see Fig. 1(d).

Next we consider the accompanying changes in the localization properties of the Floquet states by implementing the standard measures in the direct basis. Participation number is normalized by the system size, $P = 1/(N \sum |\psi_j|^4)$, so that it manifests the effective ratio of non-zero components in a Floquet vector. Their second moment, $m_2 = \sum (j - \langle j \rangle)^2 |\psi_j|^2$, where $\langle j \rangle = \sum j |\psi_j|^2$, introduces an effective width, $L = \sqrt{12m_2}/N$, also normalized, matching an effective uniform brick. Finally, we consider compactness index, $\zeta = (PN)^2/m_2$, which ranges from $\zeta \sim 0$ for sparsely populated states (a few effectively non-zero well-separated elements) to $\zeta \sim 3$ for random sets to $\zeta = 12$ for an ideally compact state (uniformly populated neighbor elements). As before, all these quantities are averaged over the period of driving.

Numerical results reveal complex trends in the localization of Floquet states, when driving amplitude starts to modify them, $A \gg 0.01$, see Fig. 2. Participation number increases among strongly localized states and decreases for quasi-extended, almost equating them when the driving is strong, $A = 1$ (Fig. 2(a)). Similar pattern is observed for the effective width of the states, except of the non-monotonous behavior for the former and retarded enhancement of the localization for the latter, see Fig. 2(b). It can be naturally explained by the theoretically predicted primary role of hybridization between strongly and quasi-extended states. Indeed, such a modification to strongly localized states should bring little to the increase of their participation number and much to their effective width; it goes vice versa for the quasi-extended states.

The complex structure of the arising Floquet states can be seen from the compactness index, which rapidly and invariably drops below $\zeta = 1$ for both kind of states (Fig. 2(c)). This reflects the hybridization process of the Floquet states and demonstrates the highly fragmented spatial structure of the states in the regime of strong driving. In this regime, relatively small participation numbers for the majority of states, $P < 0.1$, are accompanied by relatively large effective widths, $L \sim 0.3$. Local outbursts in site population numbers, scattered over the lattice, give an ultimate image of Floquet states in the direct basis under strong driving.

In conclusion, we studied the fate of the single-particle localization in a periodically-driven 1D lattice with a correlated disorder potential, a conventional model of speckled optical lattices [18]. We demonstrate that existence of the quasi-extended states, that appear due to the correlations in the potential, changes dramatically localization properties of the Floquet states. The driving-induced interaction of the states, as well as their interaction with the strongly localized states below the effective mobility edge, leads to the resonant hybridization and, finally, to the complete de-localization of the states in the basis of stationary Hamiltonian. At the same time, in the direct space we observe a trend to homogeneity: a decrease of the localization for the strongly localized states and the enhancement of localization for the quasi-extended states. Finally, in the regime of strong driving, most of the Floquet states completely loose their original structure and become spatially sparse and weakly localized. Our results can be straightforwardly generalized to the case of non-harmonic periodic driving [16, 17]. Furthermore, although the mobility edge in the used model is only an effective one (so that the quasi-extended states are not completely de-localized), our finding constitutes a step towards a much debated problem of the driven many-body localization in the presence of mobility edge [26–28].

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